Reduced Lag Moving Averages

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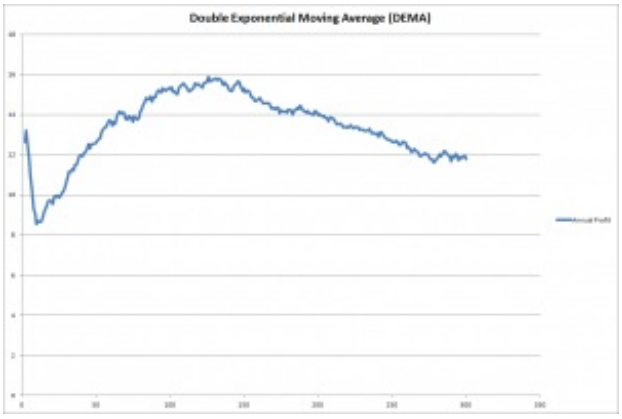
This is the fourth in a series of articles in which I am evaluating the performance of various moving average methods using a very large data sample. If this is the first time you've visited this site, I suggest you start by reading the Introduction to this study, then the posts on Weighted Moving Averages and Adaptive Moving Averages.

Averaging Methods and Results

In this post, I'll be looking at several moving average methods based on theKalman Filter, which means they attempt to compensate for lag by comparing the input and output and applying a simple method of error correction. This technique was nicknamed "twicing" by the famous mathematician John Tukey.

Double Exponential Moving Average (DEMA)

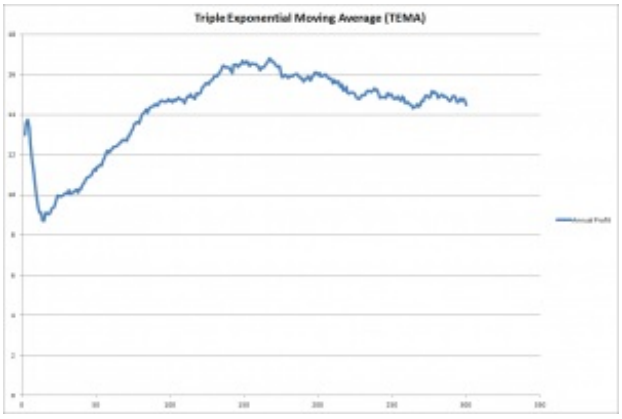


So far I've avoided posting any mathematical formulae, but this is so simple it's the best way to describe it. Here is the general formula to calculate a Double Exponential Moving Average: DEMA = EMA + EMA(price – EMA).

In other words, you take a standard Exponential Moving Average (EMA) and then add an EMA of the difference between the first EMA and the price (ie the original input). The end result has more noise than the basic EMA, but proportionally much less lag. The price we pay for this lag reduction is that sometimes the Double Exponential Moving Average will overshoot the price when it changes direction, but since we're not interested in comparing the MA with the original price anyway, this isn't a concern.

In this test, the best performing DEMA period was 126 days, which produced an average annual profit of 15.9% – significantly better than the 13.3% profit with a standard EMA.

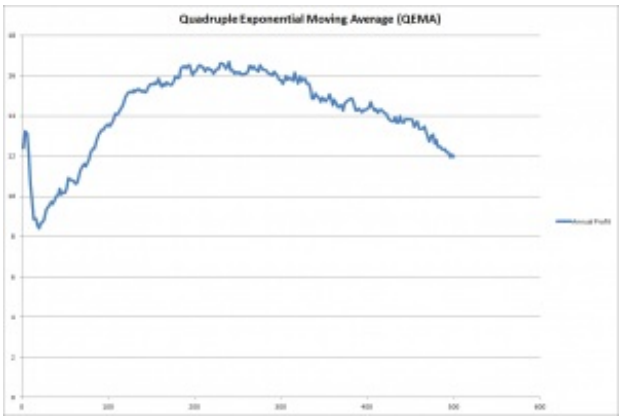
Triple Exponential Moving Average (TEMA)



As you might expect from the name, the Triple Exponential Moving Average is just a Double Exponential Moving Average that has had the error correction process applied to it a second time. So, as you might also expect, the TEMA has more noise than the DEMA for any given averaging period, but it also has proportionally even less lag and even more overshoot.

For the data being used in this study, a TEMA period of 167 days produced the highest profit, which averaged 16.8% per year.

Quadruple Exponential Moving Average (QEMA)



Given that the DEMA produced better results than the standard EMA, and the TEMA improved on these results further, it raises the question of how many times we can repeat this error correction process to get even better results.

The answer it seems is “probably none”, because the best results with a Quadruple Exponential Moving Average are slightly lower at 16.7% per year, which was achieved with an averaging period of 240 days.

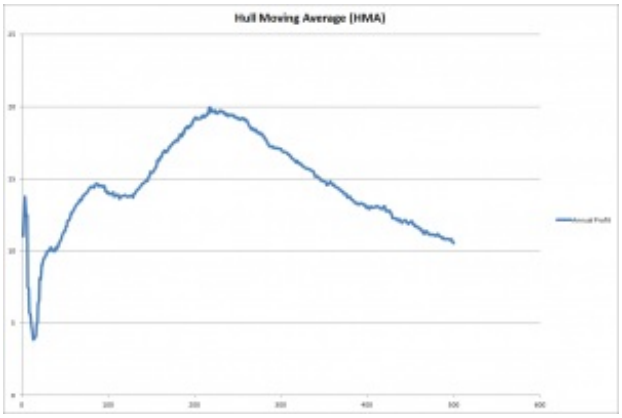
In the interests of being thorough, I’ve also tested the Quintuple, Sextuple and Septuple Exponential Moving Averages – ie with four, five and six degrees of error correction respectively. The Quintuple Exponential Moving Average actually achieved the highest annual return of 17.6% with a period of 292 days, but the graph does contain a fairly pronounced spike. So, although we can’t completely discount the value of twicing more than twice (!), it is clear that the returns of any additional error correction diminish very rapidly, and when applied to an EMA on the current data, they tail off at around 17%.

Tillson Moving Average (T3)

The Tillson Moving Average is essentially a DEMA with a less aggressive error correction setting that has been run through itself 3 times. The intended aim of this approach is to reduce or eliminate overshoot, which we're not concerned about for this study, but the additional smoothing still makes it interesting for comparison.

In this test, the highest annual return achieved by the Tillson Moving Average was 17.8%, with an averaging period of 57 days.

Hull Moving Average (HMA)



Unlike the methods described so far, which are all based on an Exponential Moving Average, the Hull Moving Average is based on a Linearly Weighted Moving Average. Also, the twicing method it uses is subtly different in that it compares the initial average with another average of half the look-back period, rather than with the original price.

Since the second average will also have roughly half the lag of the first, by adding the difference between them to the second average, we can virtually eliminate the lag. The end result is actually exactly the same as the twicing method used by the DEMA, it just requires a slightly different averaging period to get the same output.

The Hull Moving Average also removes some of the additional noise caused by the twicing process by taking a fast average of the result (using the square root of the original period).

For this test, a Hull Moving Average with a period of 218 days produced the most profit, which averaged 20% per year.

Analysis of Results

The table below show the results of the above tests in more detail, including the percentage of profitable trades and the average trade duration. I've also included the previous results for the Exponential Moving Average for comparison with the DEMA, TEMA and QEMA.

Method	Setting	Annual Profit	Avg Profit %	% Profitable	Avg Days
EMA	*87*	*13.3%*	*1.0%*	*33.5%*	*19*
DEMA	126	15.9%	1.2%	33.7%	19
TEMA	167	16.8%	1.2%	33.8%	18

QEMA	240	16.7%	1.3%	33.0%	19
Tillson	57	17.8%	7.2%	43.5%	101
Hull	218	20.0%	5.4%	36.2%	68

One interesting observation is that the twicing process has virtually no impact on the number of false signals produced by our moving averages, even though it does improve the annual profitability. This means the extra noise introduced by the error correction can be compensated for by simply increasing the averaging period, and that the extra profit must be directly attributable to the reduction in lag.

The Tillson Moving Average produced by far the highest percentage of winning trades, and given that it is also based on the DEMA, this can only be thanks to all the extra smoothing, which has removed many of the false signals. This also has the effect of increasing the average trade duration by reducing the number of signals generated.

Not only did the Hull Moving Average give the best overall profits in this test, it is also the first method so far to conclusively beat the Simple Moving Average. We can't compare it directly to the others, because it's based on a Linearly Weighted Moving Average rather than an Exponential Moving Average, but it does provide further evidence of our previous conclusions that a WMA beats an EMA and additional smoothing reduces false signals.

Conclusions

In summary, there are several conclusions we can draw from these tests. Firstly, there is no doubt that twicing is effective as a method of lag reduction, and that it can be used to improve on the accuracy of a basic moving average. Secondly, we can see that it is possible to achieve even better results by repeating the lag reduction process, but that any advantages of further repeating the process diminish very quickly. Lastly, as demonstrated by the Tillson Moving Average, although repeating the full smoothing process will increase the amount of lag again, when used in combination with a lag reduction method such as twicing, the end result can still be significantly more profitable.

We have now gathered enough evidence to very easily create our own moving average method that will outperform any of those discussed so far. However, we've still got the subjects of cyclical analysis and linear regression to get out of the way first…